Math 140 — Calculus 1
Measurable Outcomes

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Reference text: Numbers in brackets refer to sections of Stewart, *Calculus,* eighth edition Early Transcendentals

Note: Outcomes marked **(Optional)** may appear on the final exam with the unanimous consent of all instructors.

1. Review

- **1(a)** Simplify expressions involving exponents.
- **1(b)** Use logarithms to solve exponential equations.
- **1(c)** Use the properties of logarithms to simplify an expression.
- **1(d)** **(Optional)** Model a periodic phenomenon as a sine wave, determining its amplitude etc.
- **1(e)** Without a calculator, evaluate a^0, $\log_b(1)$, and $\sin\theta$ and $\cos\theta$ for $\theta = k\pi/2$ at least.

2. Slopes and rates of change

- **2(a)** Approximate the slope of $f(x)$ at a point graphically. [2.1]
- **2(b)** Approximate the slope of $f(x)$ at a point using a difference quotient. [2.1]
- **2(c)** Find the average rate of change of a function from a table. [2.1]
- **2(d)** Interpret the slope of a function as a rate of change / sensitivity, including units. [2.1]

3. Limits

- **3(a)** Approximate a limit using a table. [2.2]
- **3(b)** Distinguish between a finite limit, infinite limit, and no limit using a table. [2.2]
- **3(c)** Approximate limits (including one-sided limits) from a graph. [2.2]

3(d) Understand that $\lim_{x \to a} f(x)$ does not depend on $f(a)$. [2.2]

3(e) Determine the value of a limit given information about the corresponding one-sided limits. [2.2]

3(f) **(Optional)** Use the Squeeze Theorem to evaluate a limit. [2.3]

3(g) Find the limit of common functions at ∞ and $-\infty$. [2.6]

3(h) Find the limit of a ratio of functions at ∞ or $-\infty$ using formal algebraic manipulations. [2.6]

3(i) Find the limit of a ratio of functions at ∞ or $-\infty$ using informal reasoning about "lower-order terms". [2.6]

4. Continuity

4(a) Evaluate a limit using continuity. [2.3, 2.5]

4(b) Evaluate the limit of a rational function by simplifying and then using continuity. [2.3, 2.5]

4(c) Evaluate one-sided limits of a piecewise continuous function using continuity. [2.3, 2.5]

4(d) Given that $f(x)$ satisfies certain conditions, determine what other conditions are necessary in order for $f(x)$ to be continuous. [2.5]

4(e) Determine whether a given piecewise function is continuous at a point. [2.5]

4(f) Determine the location and type of discontinuities of $f(x)$ on $[a, b]$. [2.5]

4(g) Use the Intermediate Value Theorem to write an argument that proves that a function has a root on a given interval. [2.5]

4(h) Critique a given argument that uses the Intermediate Value Theorem. [2.5]

4(i) **(Optional)** Use the Bisection Method and the Intermediate Value Theorem to approximate the root of a function to a given degree of accuracy. [2.5]

5. The Derivative

5(a) Evaluate the derivative of a function at a point using the limit definition. [2.7]

5(b) Sketch a rough graph of $f'(x)$ given a graph of $f(x)$. [2.8]

5(c) Determine at which points a function is not differentiable, given its graph. [2.8]

5(d) Interpret the meaning of $f'(a)$ as a rate of change / sensitivity. [2.7, 2.8]

6. Derivative formulas

6(a) Distinguish constants from non-constants. [3.1]

6(b) Distinguish exponential functions from power functions. [3.1]

6(c) Rewrite functions as powers of x when appropriate ($1/x$, $\sqrt{x}$). [3.1]

6(d) Implicitly use linearity when taking derivatives. [3.1]

6(e) Take the derivatives of polynomials and power functions. [3.1]

6(f) Take the derivative of exponential functions. [3.1]

6(g) Take the derivative of trigonometric functions. [3.3]

6(h) Take the derivative of logarithmic functions. [3.6]

6(i) Use the product rule when appropriate. [3.2]

6(j) Use the quotient rule when appropriate. [3.2]

6(k) Distinguish between function composition and function multiplication. [3.4]

6(l) Use the chain rule when appropriate. [3.4]

6(m) Use implicit differentiation when appropriate. [3.5]

6(n) **(Optional)** Use logarithmic differentiation to find the derivative of $f(x)^{g(x)}$. [3.6]

7. Applications of Derivatives

7(a) Given the position function of an object, determine: [3.7]

- Its velocity
- Its acceleration
- When the object is moving up/down (forward/backward)
- When the object is speeding up
- The net distance traveled over a time interval
- The total distance traveled over a time interval

7(b) Solve an abstract related rates problem. (e.g. If $y = f(x)$, find dy/dt when $x = 3$ and $dx/dt = 5$.) [3.9]

7(c) Model a physical situation as a related rates problem. [3.9]

7(d) Use the derivative to write the equation of a tangent line. [3.10]

7(e) Use the tangent line / linearization to approximate a function's value at another point. [3.10]

7(f) **(Optional)** Use differentials to approximate how an error in x affects the error in $f(x)$. [3.10]

7(g) Write a logical argument that uses Rolle's Theorem to prove that a function has at most one root on a given interval. [4.2]

7(h) **(Optional)** Critique a flawed logical argument that misuses Rolle's Theorem. [4.2]

7(i) Use the Mean Value Theorem to bound $f(b)$ given information about $f(a)$ and about $f'(x)$ on $[a,b]$. [4.2]

7(j) Model a physical situation in a way that it generates a problem like the above. [4.2]

8. Optimization

8(a) Find the local and absolute maxima and minima of a function from its graph. [4.1]

8(b) Determine the critical points of a function. [4.1]

8(c) Find the absolute maximum and minimum of a continuous function on a closed interval. [4.1]

8(d) Solve an abstract optimization problem with a two variable function and a constraint. [4.7]

8(e) Model and solve a physical optimization problem with a two variable function and constraint. [4.7]

9. Graphing with calculus

9(a) Determine where a graph is increasing / decreasing by analyzing the sign of $f'(x)$. [4.3]

9(b) Determine where a graph is concave up / concave down by analyzing the sign of $f''(x)$. [4.3]

9(c) Find the inflection points of a function. [4.3]

9(d) Use the First Derivative Test to classify a critical point as a local maximum, local minimum, or neither. [4.3]

9(e) Use the Second Derivative Test to classify a critical point as a local maximum, local minimum, or indeterminate. [4.3]

9(f) Sketch a piece of a graph with a given sign of f' and f''. [4.3]

9(g) Given a graph, determine the intervals where f' is positive and negative. [4.3]

9(h) Given a graph, determine the intervals where f'' is positive and negative. [4.3]

9(i) Sketch the graph of a polynomial using information about f' and f''. [4.5]

9(j) Find the location of the vertical asymptotes of a function. [2.2, 4.5]

9(k) Find the location of the horizontal asymptotes of a function. [2.6, 4.5]

9(l) Sketch the graph of a rational function using asymptotes and information about f'. [4.5]

10. Antiderivatives

10(a) Calculate the antiderivative of a constant. [4.9]

10(b) Calculate the antiderivative of powers of x, including $1/x$. [4.9]

10(c) Calculate the antiderivative of exponential functions. [4.9]

10(d) Calculate the antiderivative of $\sin x$ and $\cos x$. [4.9]

10(e) Implicitly use linearity when calculating antiderivatives. [4.9]

10(f) Solve the initial-value problem $dy/dx = f(x)$, $y(a) = b$. [4.9]

10(g) Find the position of an object given its velocity function and one initial value. [4.9]

10(h) **(Optional)** Find the position of an object given its acceleration function and two initial values. [4.9]

11. Areas, Riemann Sums, and Definite Integrals

11(a) Approximate the area under a curve using a graph and a specified strategy. [5.1]

11(b) Approximate the accumulated change using a table of the rate of change. (e.g. estimate distance traveled given a table of velocity) [5.1]

11(c) Approximate the area under a curve using a Riemann sum, given a formula for $f(x)$. [5.1]

11(d) Evaluate a definite integral of a function using its graph and basic plane geometry (area of triangles, quadrilaterals, portions of circles). [5.2]

11(e) Approximate a definite integral using a Riemann sum with specified parameters. [5.2]

12. Fundamental Theorem of Calculus

12(a) Evaluate a definite integral of a linear combination of basic functions using the Fundamental Theorem of Calculus. [5.3]

12(b) Find the derivative of a function defined in terms of an integral, using the Fundamental Theorem of Calculus. [5.3]

12(c) Evaluate an indefinite integral of a linear combination of basic functions. [5.4]

12(d) Evaluate an indefinite integral of a product or quotient of functions by simplifying the integrand first. [5.4]

12(e) Interpret a definite integral in terms of accumulated change. [5.4]

12(f) Evaluate an indefinite integral in the form $\displaystyle\int cf(g(x)) \cdot g'(x)\ dx$ using a substitution. [5.5]

12(g) Evaluate a definite integral in the form $\displaystyle\int cf(g(x)) \cdot g'(x)\ dx$ using a substitution. [5.5]

12(h) Evaluate an indefinite or definite integral using a 'linear shift' substitution. (e.g. $\int x^2\sqrt{2x+3}\ dx$). [5.5]

13. Applications of Integrals

13(a) **(Optional)** Find the points of intersection of two curves, such as a line and parabola, two parabolas, or two functions of the form cx^n. [6.1]

13(b) **(Optional)** Find the area between curves using a definite integral. [6.1]

13(c) **(Optional)** Find the volume of a solid of revolution using disks or washers. [6.2]